

## OUTER-PLANET MISSION ANALYSIS USING SOLAR-ELECTRIC ION PROPULSION

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Outer-planet mission analysis was performed using three next generation solar-electric ion thruster models. Optimal trajectories are presented that maximize the delivered mass to the designated outer planet. Trajectories to Saturn and Neptune with a single Venus gravity assist are investigated. For each thruster model, the delivered mass versus flight time curve was generated to obtain thruster model performance. The effects of power to the thrusters and resonance ratio of Venutian orbital periods to spacecraft period were also studied. Multiple locally optimal trajectories to Saturn and Neptune have been discovered in different regions of the parameter search space. The characteristics of each trajectory are noted.

### INTRODUCTION

After the success of the Deep Space 1 mission, Solar Electric Propulsion Systems (SEPS) have entered the mainstream of propulsion system candidates for various missions. Through their long-duration, high-efficiency operation, SEPS capabilities allow new ways to explore the inner and outer solar system, enabling missions that can be difficult and expensive to reach with chemical propulsion systems.

In this paper, Saturn and Neptune are considered as the potential targets of SEPS missions. At 9.5 AU from the Sun, Saturn and one of its satellites, Titan, have been the target of previous interplanetary missions due to scientific interest in the dense atmosphere of Titan and possible presence of water<sup>1</sup>. SEPS can deliver significant mass to Titan for various scientific missions. Neptune has also been considered as potentially possessing water and thus has had strong scientific attraction, but at 30 AU from the Sun, it has been very difficult to explore<sup>2</sup>. In this study, multiple optimal trajectories were generated using SEPTOP (Solar Electric Propulsion Trajectory Optimization Program)<sup>3</sup>. With given initial conditions, SEPTOP calculates a trajectory that maximizes the delivered mass to a destination. That delivered mass may include the scientific payload along with aero-capture equipment, propulsion system and supporting bus.

In exploration of outer solar system bodies like Saturn or Neptune, a planetary gravity assist (GA) has commonly been used since one or more GAs have the potential to save propellant, reduce flight time, or both. Because of these advantages, many previous interplanetary missions (for example, Mariner 10, Voyager I, II, Galileo, Cassini and NEAR) exploited the GA<sup>4</sup>. This technique is again used in this study, employing a single Venus GA to generate the Earth-Venus-Saturn (EVS) and Earth-Venus-Neptune (EVN) SEPTOP trajectories presented within.

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## SOLAR ELECTRIC PROPULSION SYSTEM

Near-term next generation ion propulsion systems were used as the primary spacecraft propulsion source for the outer-planet missions examined. Three different models were investigated and compared for their performance, propellant usage, and throughput: High-Thrust-To-Power (HTTP)  $I_{sp}$  3900 sec, High- $I_{sp}$ -To-Power (HITP)  $I_{sp}$  3900 sec, and HITP  $I_{sp}$  4070 sec<sup>5</sup>. The  $I_{sp}$  values used in these engine descriptions are the values at each respective engine's maximum operating power ( $P_{max}$ ) of 6.1 kW. The minimum operating power ( $P_{min}$ ) is 1.1 kW for all three thruster models. The thrust and mass flow rate for the thruster models are given in Patterson et al<sup>5</sup>. According to this reference, the lower  $I_{sp}$  (HTTP  $I_{sp}$  3900 sec) thruster has the largest thrust and mass flow rate for power levels into the thruster less than  $P_{max}$ . The power into the thrusters is generated by solar arrays and processed by Power Process Units (PPU). The power generated by the solar arrays ( $P_0$  at 1 AU from the Sun) is inversely proportional to the square of the distance between the spacecraft and the Sun as shown in Figure 1. Here  $P_0$  at 1 AU is 30 kW. PPU efficiency is less than 100% (varying as a function of PPU input power), which results in losses when processing power from the solar array. Power to the thrusters is provided by the PPU, therefore the power generated by the solar array needs to be greater than the number of thrusters multiplied by  $P_{max}$  if multiple thrusters are to be operated at maximum power. The performance of the thruster model and  $P_0$  will be analyzed at the later part of this paper.

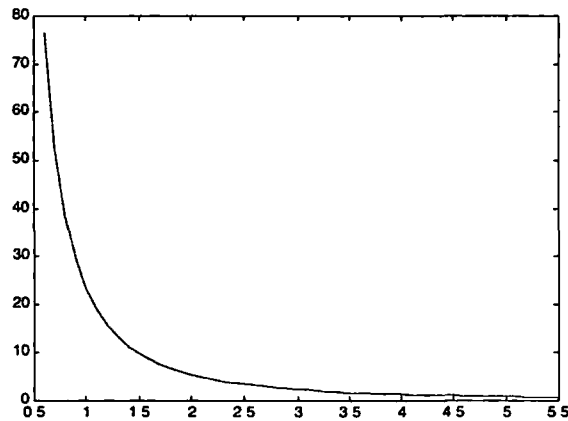


Figure 1 Power profile of solar array

## VENUS GRAVITY ASSIST (VGA)

A gravity assist will alter the orbital characteristics of an interplanetary spacecraft. Typically a change in spacecraft kinetic energy is targeted. The heliocentric kinetic energy can be increased or decreased depending on the angles between the velocity vector of the flyby planet and the hyperbolic excess velocity vectors of the spacecraft<sup>6</sup>. Figure 2 illustrates the mechanism of a GA.  $V_p$  is the velocity vector of the flyby planet in heliocentric coordinates.  $V_{\infty I}$  and  $V_{\infty O}$  are the respective incoming and outgoing hyperbolic excess velocity of the spacecraft in flyby-planet-centered coordinates. By examining Figure 2, it is easy to see that a flyby that has  $|\alpha| < |\alpha + \beta|$  can only provide heliocentric velocity gain. In other words, a spacecraft should pass behind the flyby planet in the direction of planet's orbital motion to gain heliocentric velocity.

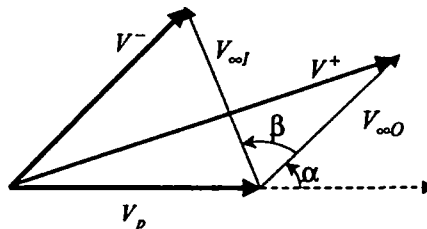


Figure 2 Mechanism of a gravity assist

The amount of velocity gain,  $\Delta V$ , can be attained as a function of the magnitude of hyperbolic excess velocity,  $V_\infty$ , flyby planet radius,  $r_s$ , and the periapse radius of the flyby hyperbola,  $r_p$ . The function for  $\Delta V_{flyby}$  is shown in Eq. 1. This equation was used to compute the  $\Delta V_{flyby}$  of an optimal solution<sup>6</sup>.

$$\frac{\Delta V_{flyby}}{V_s} = \frac{2V_\infty / V_s}{1 + \left(\frac{V_\infty}{V_s}\right)^2 \left(\frac{r_p}{r_s}\right)} \quad (1)$$

## OPTIMIZATION

The trajectory optimization problem with variable thrust and thrust direction has been previously investigated<sup>3,7</sup>. The problem can be formulated to optimize a number of parameters, but in this research the final mass delivered to Saturn or Neptune is maximized.

SEPTOP was used for the mission analysis of Deep Space 1 at the Jet Propulsion Laboratory. SEPTOP is a two-body, Sun-centered, low-thrust trajectory optimization program for preliminary mission feasibility studies, but it also provides relatively accurate performance estimates. The program determines a numerical solution to a two point boundary value problem that satisfies intermediate boundary constraints. In SEPTOP, the user estimates initial conditions and a shooting method is then used to integrate the trajectory from an initial time to final time. An error is computed at the final time and used to correct the estimate of the initial conditions. This process is repeated until the error is reduced to within the prescribed tolerance<sup>8</sup>. The required inputs are therefore flight time, nominal epoch,  $P_0$ , flyby radius, and launch vehicle specifications. SEPTOP can model variable thrust and mass flow rate as a function of power into the PPU. The power generated from a solar array is modeled as a function of the spacecraft's distance from the Sun. Thruster and solar array models are therefore also required as inputs.

With these inputs, many parameters are free to be selected. For example, initial values for Lagrange multipliers, a launch date, launch energy ( $C_3$ ), and flyby date to maximize the delivered mass to the destination. The solution from SEPTOP is the local optimal solution in the parameter space. It is therefore possible for multiple solutions to exist with similar inputs. The characteristics of such solutions will be explained with the example of a Neptune mission. The flight time is one of the main mission design drivers. Determining the minimum flight time trajectory that delivers a specified mass is commonly of interest.

## TRAJECTORY CLASSIFICATION AND ANALYSIS

All optimal trajectories within this paper are categorized by their characteristics. Trajectories are first categorized by launch date. In some situations, two local optimal trajectories with the similar inputs exist, but one has an early launch date (early type) and the other has a later launch date (late type). Second, the trajectories are categorized by their resonance ratio. The resonance ratio is the number of Venus revolutions for one revolution of a spacecraft around the Sun. For instance, a 3:1 resonance ratio is one where roughly three Venus years occur during the period from launch to flyby for a spacecraft. In this paper, the performance of trajectories for each launch date type and resonance ratio is investigated, as well as the performance of thruster and power system models.

Delivered mass is first investigated for three thruster models in order to find the best performing thruster model. The delivered mass versus flight time for a late type, 4:1 resonance ratio, Saturn mission using three thruster models is shown in Figure 3. Performance of a trajectory is defined as the mass delivered for a given flight time. Figure 3 shows the performance of three different thruster models. Among the three models, the HTTP  $I_{sp}$  3900 sec model shows the best performance. It will therefore be used for further analysis in this paper. This is also true for the Neptune mission (Figure 4), so the HTTP  $I_{sp}$  3900 sec model will also be used as a basis for further Neptune mission analysis. Since the results of the Saturn and Neptune mission analysis are similar in trend, only one representative result for each mission is given in this paper. In all examples presented, the launch vehicle used is the Delta IV (4,2+) Its launch capacity is about 6000 kg to geo-synchronous transfer orbit with a faring diameter of 4 m<sup>9</sup>.

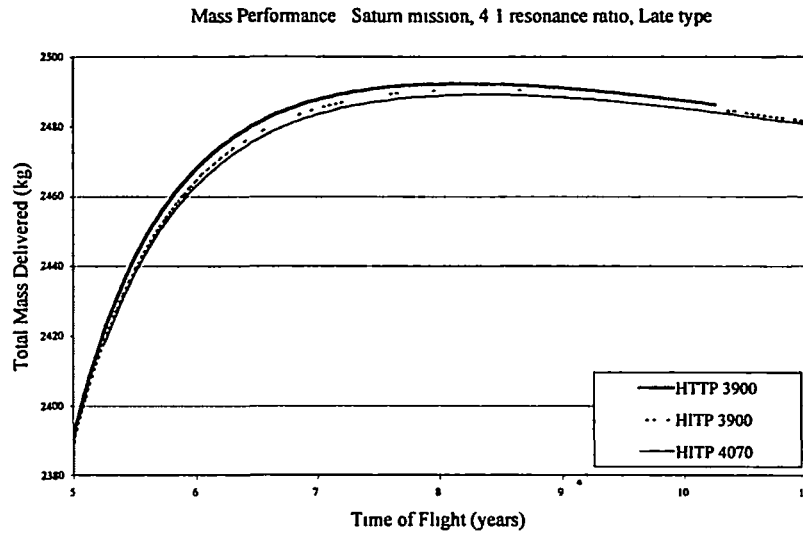


Figure 3 Payload Mass vs Flight Time Saturn mission, 4:1 resonance ratio, late type trajectory

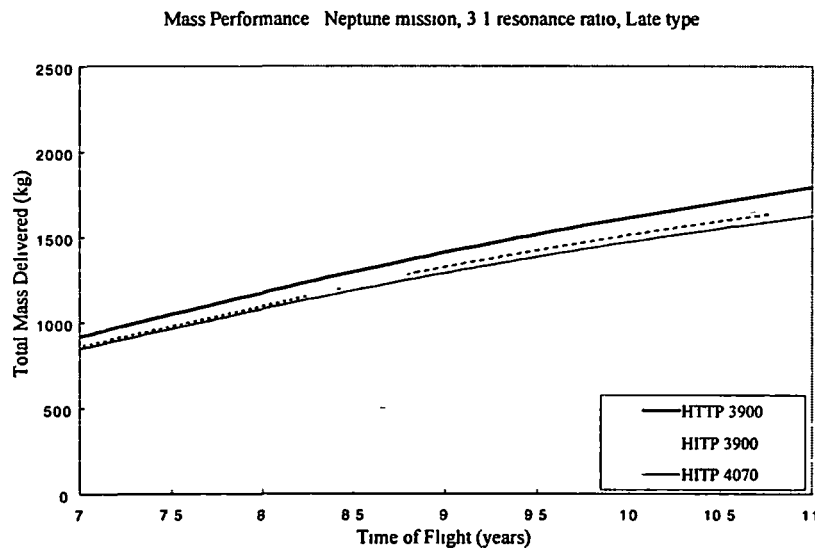


Figure 4 Payload Mass vs Flight Time. Neptune mission, 3:1 resonance ratio, late type trajectory

A  $P_0$  variation study was executed for a Saturn and a Neptune missions. Results are shown in Figure 5. This result provides a design reference for the solar array sizing. If an optimal  $P_0$  to deliver the most payload mass exists, it can be found once the power, propulsion and bus sizing is computed for the range of  $P_0$ . A companion paper discusses this result<sup>10</sup>. Figure 5 also shows that Saturn can be reached with less  $P_0$  for a given delivered mass than Neptune. Two trajectories for a Neptune mission that have the same flight time, type, and resonance ratio but different  $P_0$  are shown in Figure 6. In this figure, the trajectories are almost the same, but the thrusting phase is longer in the higher  $P_0$  trajectory than the lower  $P_0$  trajectory. This is due to the fact that in the high  $P_0$  trajectory, the spacecraft has power available to it – greater than the minimum required to operate the thrusters – at farther distances from the Sun. This difference results in the higher  $P_0$  trajectory delivering more payload mass to the destination since a lower  $C_3$  is required by the launch vehicle. When given the option, the trajectory optimizer generally chooses to use the higher efficiency SEPS over the less efficient launch vehicle to provide energy for the trajectory. If the higher efficiency

SEPS is made available for a longer period of time due to higher reference power,  $P_0$ , the result is generally increased thrusting with SEPS and a reduction in  $C_3$  provided by the launch vehicle.

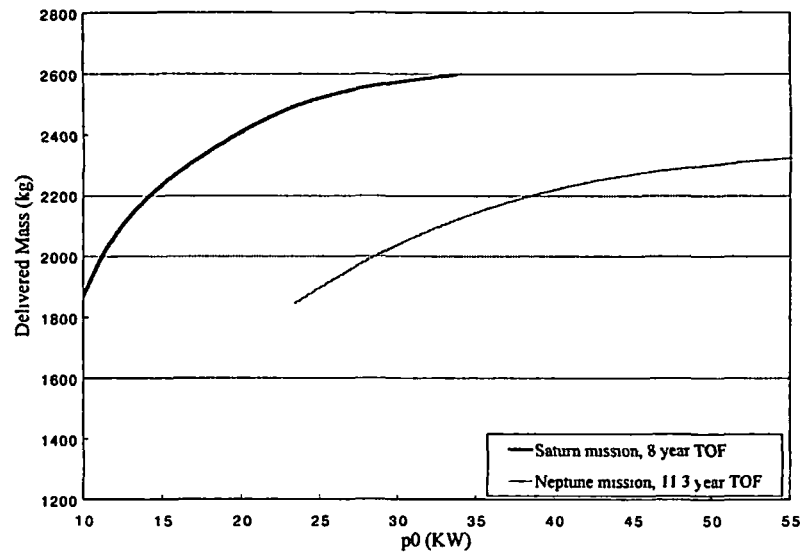


Figure 5 Payload Mass vs  $P_0$  Variation: Saturn mission (8 year flight time, 4:1 resonance ratio) and Neptune mission (11.3 year flight time, 3:1 resonance ratio)

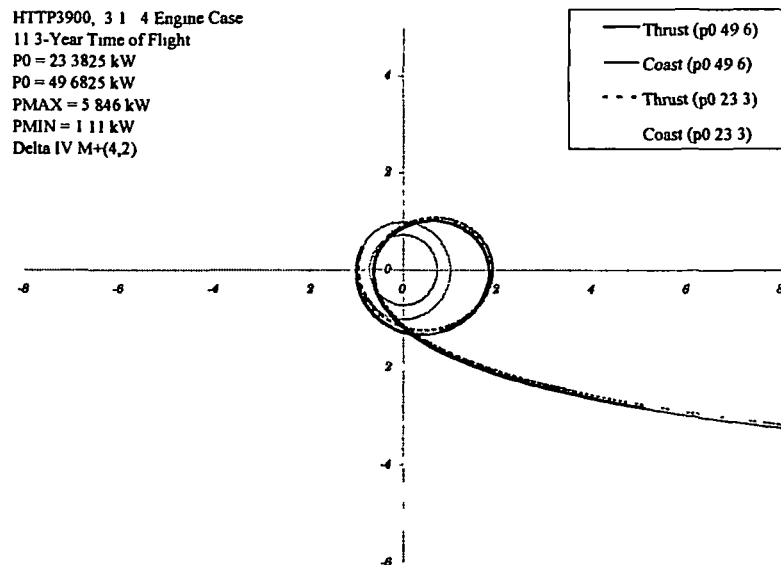


Figure 6 Trajectory plots for two  $P_0$  levels (Neptune mission, 11.3 years flight time, 3:1 resonance ratio)

In Figure 7, two trajectories with the same flight time (11.3 year) and resonance ratio (3:1) but different launch dates are shown for a Neptune mission. Since their performances are almost the same, this indicates that there are two nearly equivalent launch opportunities. The late type trajectory may deliver slightly less mass, but it uses less on-board propellant and therefore will be used as default trajectory type for further analysis.

HTTP3900 3 1, Early & Late Type 4 Engine Case  
 11.3-Year Time of Flight  
 $P_0 = 23\,382.5\text{ kW}$  (Power into Thrusters)  
 Total Mass Delivered Late = 1844.58 kg  
 Total Mass Delivered Early = 1850.58 kg  
 Propellant Mass Late = 955.24 kg ( $\Delta V = 14.8\text{ km/s}$ )  
 Propellant Mass Early = 1023.46 kg ( $\Delta V = 13.4\text{ km/s}$ )  
 $C_3\text{ Late} = 12.30\text{ km}^2/\text{sec}^2$   
 $C_3\text{ Early} = 11.16\text{ km}^2/\text{sec}^2$   
 $\Delta IV\text{ M}+(4,2)$

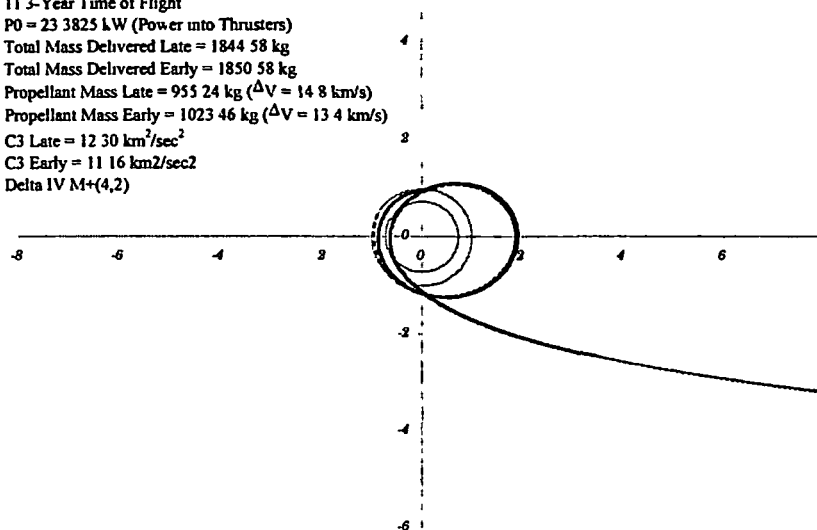


Figure 7 Trajectory plots for two launch dates (Neptune mission, 11.3 years flight time, 3:1 resonance ratio)

Because of the flexibility of SEPS in mission design, there are possible launch dates between the early and the late launch dates. Figure 8 shows the performance variation for trajectories with the same flight time (8.5 year) at various launch dates. According to the figure, the launch window is very flexible with less than 10 kg of performance penalty between the early and the late launch date. For the trajectories between the two fully optimal solutions, the launch date is not optimized but given as an input.

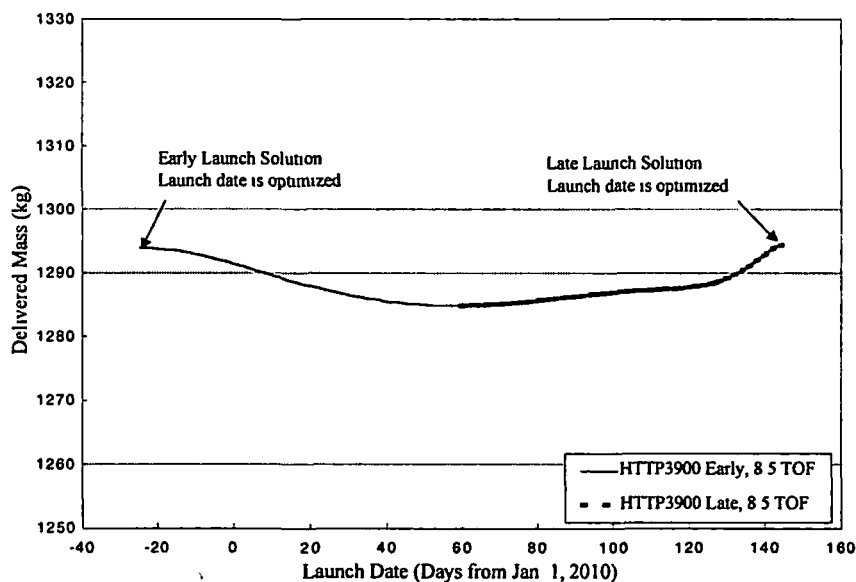


Figure 8 Delivered Mass vs Launch Date. Neptune mission, HTTP 3900 sec., 8.5 years, 3:1 resonance ratio

Figure 9 shows three trajectories for a Neptune mission with three different resonance ratios. The flight time of all trajectories in the figure is 9.6 years. It is clear that a spacecraft on the trajectory with the largest resonance ratio spends the most time thrusting before the flyby, so it may deliver more mass than a trajectory that has a smaller resonance ratio. But the larger resonance ratio trajectory needs more launch energy. More launch energy means less on-board propellant and a larger proportion of the total trajectory energy being provided by an inefficient launch.

vehicle rather than the highly efficient low-thrust engine. Trade-offs between the launch energy and the propellant mass result in an optimal resonance ratio of 4:1 for this 9.6 year mission.

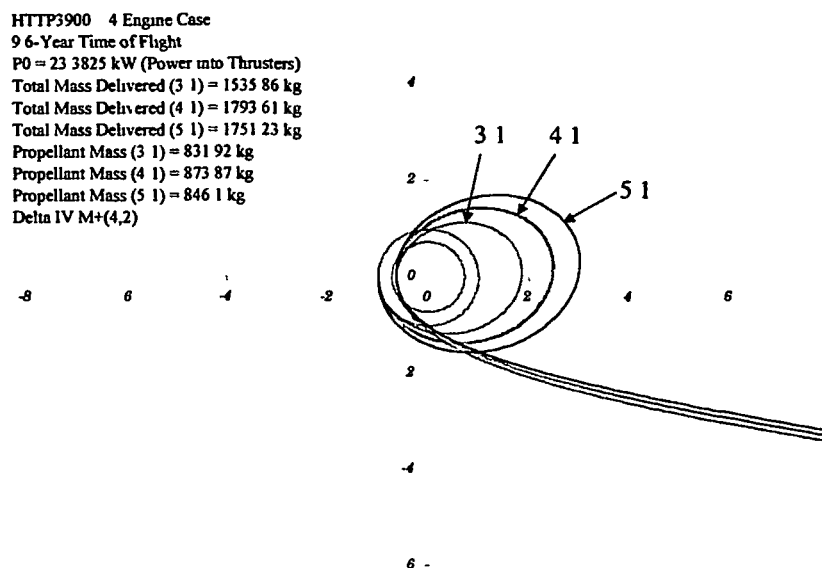


Figure 9 Trajectory plots of three different resonance ratio trajectories for a Neptune mission

Table 2 shows detailed trajectory data. The cut-off date is the date when the final thrust phase ends. The cut-off velocity is the heliocentric velocity of the spacecraft at this same date. The launch velocity is the heliocentric velocity of a spacecraft when it has just separated from the launch vehicle. For all times of flight, the launch velocity is higher for higher resonance ratio trajectories because greater launch energy is required to follow the larger trajectory energy. Similarly, the cut-off date is later and cut-off velocity higher for higher resonance ratio trajectories because in these higher resonance ratio trajectories there is less time between the cut-off date and the arrival date. The cut-off velocity therefore needs to be higher to complete the mission in the given flight time. Also, the 3:1 ratio trajectories have the lowest launch velocities for shorter and longer times of flight, while the 4:1 trajectories are lowest for the intermediate flight times. This result can be correlated with launch  $C_3$  and is discussed later on.

Table 2 Detailed Characteristics of Earth-Venus-Neptune Resonance Ratio Trajectories

Time of flight (years)	Resonance Ratio	Launch Date	Cut-off Date	Arrival Date	Launch Velocity (km/sec)	Cut-off Velocity (km/sec)
7	3:1	June 20, 2010	May 13, 2012	June 20, 2017	33.6	29.5
	4:1	June 5, 2010	Dec 11, 2012	June 5, 2017	34.6	32.2
	5:1	June 9, 2010	July 11, 2013	June 8, 2017	34.9	35.6
9.6	3:1	May 7, 2010	June 17, 2012	Dec 13, 2019	33.8	23.8
	4:1	Mar. 27, 2010	Jan 19, 2013	Nov 2, 2019	33.6	25.0
	5:1	Mar 12, 2010	Mar 22, 2013	Oct 17, 2019	33.9	26.3
15	3:1	Mar 7, 2010	July 19, 2012	Mar 6, 2025	32.5	20.2
	4:1	Feb 11, 2010	Sept 1, 2012	Feb 10, 2025	33.2	28.6
	5:1	Feb 10, 2010	Feb 14, 2013	Feb 10, 2025	33.8	36.3

Figure 10 shows the performances of the three resonance ratio trajectories for a Neptune mission. At a 7.2 year time of flight, the performances of the 3:1 and 4:1 trajectories coincide, while at an 8.05 year time of flight the performance of 3:1 and 5:1 trajectories coincide. Smaller resonance ratio trajectories are superior in short flight

time missions due to the fact that larger resonance ratio trajectories have to spend longer periods of time in flight before the flyby so they have very little time after the flyby to reach their destination. This results in the larger resonance ratio trajectory having lower performance than smaller resonance ratio trajectories in short flight time missions.

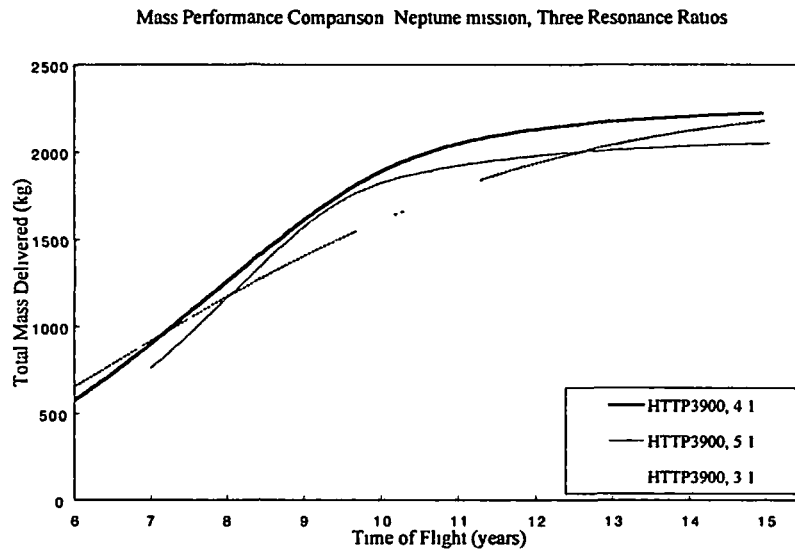


Figure 10 Payload Mass vs Flight Time Neptune mission, resonance ratio comparison

The performance of 3:1 trajectories is similarly superior in longer flight time missions compared to 4:1 and 5:1 trajectories, though for different reasons. In Figure 11, the launch  $C_3$  plots for several Neptune missions are shown. For a given resonance ratio, there is an optimum launch energy for each time of flight. This launch  $C_3$  is generally larger for larger resonance ratio trajectories, though not always, as seen in Figure 11. This fact causes the larger resonance ratio trajectories to have poorer performance for longer flight time missions. For a Neptune mission, the 4:1 trajectories are the best performing trajectories for intermediate flight time (7 ~ 15 years), but in general the best performing resonance ratio will be determined by the desired flight time, the flyby and destination planets, and the characteristics of the SEPS.

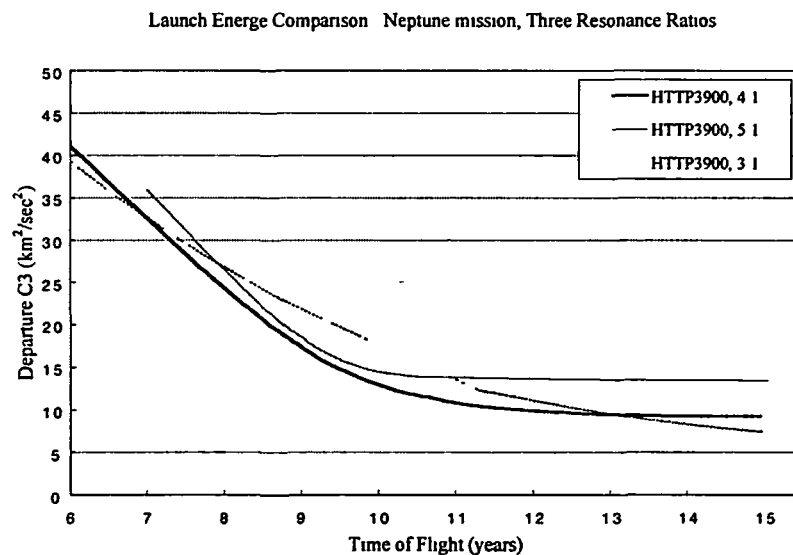


Figure 11 Launch Energy vs Flight Time Neptune mission, resonance ratio comparison



Table 3 shows detailed trajectory characteristics for a Saturn mission. The results are similar to Table 2 in that, for a Saturn mission, launch velocity is higher for higher resonance ratio trajectories, cut-off date is later for higher resonance ratio trajectories, and cut-off velocity is higher for higher resonance ratio trajectories for all times of flight.

Table 3 Detailed Characteristics of Earth-Venus-Saturn Resonance Ratio Trajectories

Time of flight (years)	Resonance Ratio	Launch Date	Cut-off Date	Arrival Date	Launch Velocity (km/sec)	Cut-off Velocity (km/sec)
2.95	3:1	Mar 3, 2011	Nov 11, 2012	Feb. 13, 2014	32.8	25.3
	4:1	Mar 20, 2011	May 3, 2013	Mar. 2, 2014	34.6	35.3
	5:1	May 20, 2011	Sept 1, 2013	May 2, 2014	34.6	56.3
4.6	3:1	Dec 8, 2010	Jan 21, 2013	July 15, 2015	32.7	17.8
	4:1	Nov 8, 2010	Apr. 7, 2013	June 15, 2015	32.8	24.5
	5:1	Nov 27, 2010	Nov 1, 2013	July 4, 2015	33.8	28.2
6	3:1	Nov 1, 2010	Feb 14, 2013	Oct 31, 2016	31.3	16.2
	4:1	Oct 21, 2010	Feb 22, 2013	Oct. 21, 2016	32.4	25.7
	5:1	Oct 30, 2010	July 23, 2013	Oct 30, 2016	33.3	36.0

Saturn missions show similar results in a resonance ratio analysis. Figure 12 shows the performances of the three resonance ratio trajectories and Figure 13 shows the launch energy comparison of the three resonance ratio trajectories for a Saturn mission. The reasons that the 4:1 resonance ratio trajectories are superior to 3:1 trajectories for intermediate flight time (4 ~ 5.5 years) are similar to the Neptune case, but the performance difference between 3:1 and 5:1 trajectories is different from that for Neptune missions. The reason for this difference between Saturn and Neptune missions is the different total velocity increment (launch energy + on-board thrust energy) in each mission. A Saturn mission requires less velocity increment than a Neptune mission, but the launch energy to make a resonance ratio trajectory does not differ significantly for a given time of flight. Therefore, for a Saturn mission, the launch energy is a higher percentage of the total velocity increment than it is for a Neptune mission, especially in high resonance ratio missions. This phenomenon produces 5:1 trajectories that consistently perform worse than 3:1 trajectories in a Saturn mission.

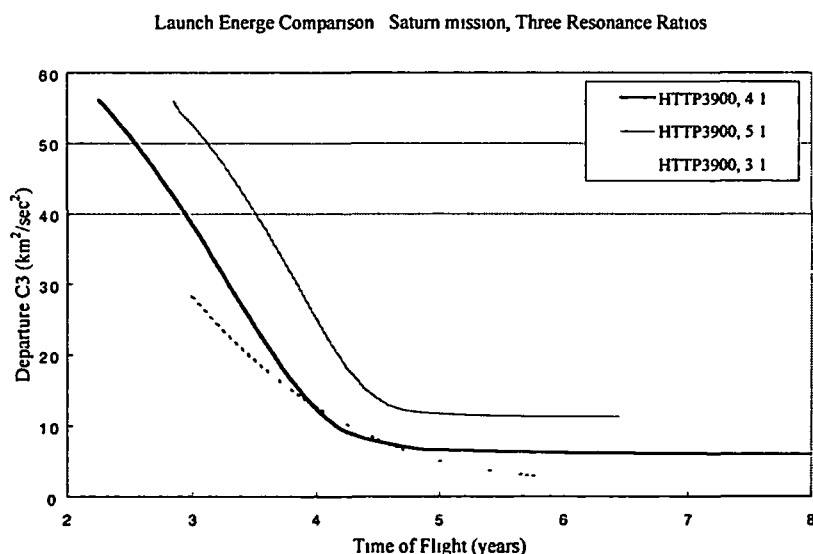


Figure 12 Payload Mass vs Flight Time Saturn mission, resonance ratio comparison

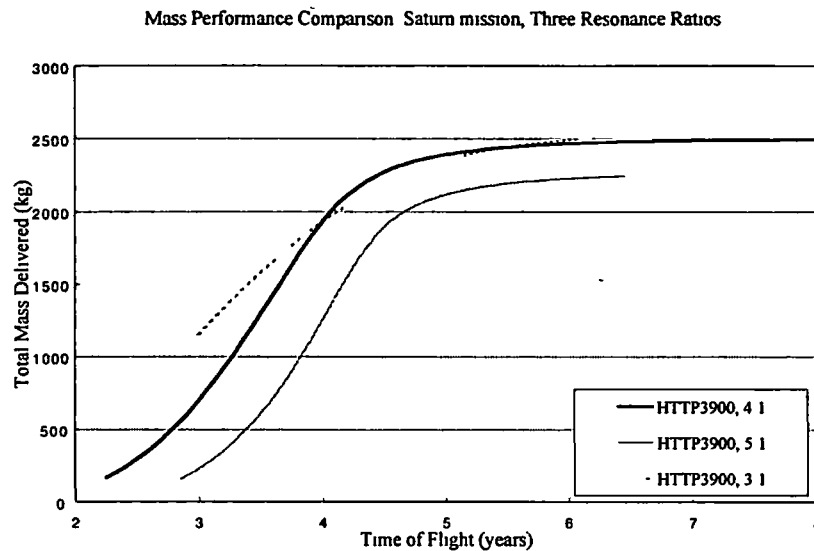


Figure 13 Launch Energy vs Flight Time Saturn mission, resonance ratio comparison

In designing an actual mission trajectory, the total operation time of the SEPS should be considered since there is a limitation in total operation time in current SEPS design and the required operation time may be longer than the maximum for the state-of-the-art thruster design<sup>11</sup>.

## CONCLUSION

Optimal trajectories are investigated with three near-term solar electric ion propulsion thrusters. Multiple optimal trajectories are generated and their characteristics are analyzed. For the best performance of the missions considered here (the largest delivered mass at the destination), the higher thrust and mass flow rate thrusters (lowest  $I_{sp}$  thrusters) are slightly superior. An optimal resonance ratio for a given mission was also discovered. The performance difference between the early and late launch type is not significant, and a larger launch window exists that delivers consistent performance. The performance along with the power variation result will be used for solar array and spacecraft sizing to determine the scientific payload mass. Finally, this paper yields total mass estimates that can be delivered within a given mission time with developing SEPS technology.

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